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Natural convection and conduction in enclosures with multiple vertical partitions

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Abstract—Laminar natural convection and conduction in enclosures having multiple partitions with finite thickness and conductivity are studied theoretically. The governing equations are solved using the finite difference formulation and volume control method. The study covers Ra from 10^3 to 10^7, $Pr = 0.72$ (air), aspect ratio $A = H/W$ from 5 to 20, $B = w/W$ from 0.1 to 0.9 and $C = l/W$ from 0.01 to 0.1. The partition number N is varied from 1 to 5 and the thermal conductivity ratio of partition to fluid k_r from 1 to 10^4. The results are reduced in terms of Nu as a function of Ra and various other parameters.

INTRODUCTION

IN MANY practical cases, enclosures with vertical partitions are used to modify heat transfer by natural convection, conduction or radiation. A literature review shows that there have been various theoretical and experimental studies on natural convection in enclosures with single and multiple diathermal partitions: (i) studies with a single partition in vertical enclosures [1, 2] and in inclined enclosures [3] and (ii) studies with multiple partitions in vertical enclosures including double partitions [4] and multiple partitions [5, 6]. The boundary conditions in these studies consisted of two isothermal vertical bounding walls and two adiabatic horizontal bounding walls with equally spaced partitions. The effect of an off-center partition on natural convection heat transfer has also been reported by Nishimura *et al.* [7]. The results of earlier studies show that the heat transfer rates through the system with partitions decrease with an increasing number of partitions.

Anderson and Bejan [4] studied enclosures with a single partition analytically based on the Oseen linearization method. The study was in the boundary layer regime and the effect of the conductance through the partition was supposed to be negligible. They confirmed their results experimentally using an enclosure with a double partition. The experimental results were correlated to obtain a relation of heat transfer between the two ends. It was proportional to $(1+N)^{0.61}$ where N is the number of partitions. Nishimura *et al.* [6, 7] used a boundary layer solution and confirmed their results experimentally for the case of an enclosure with an off-center partition as well as with equally spaced multiple partitions. They also extended their study to a non-boundary layer regime by a numerical study. They showed that the isothermal partition model was not suitable for multiple partitions and derived a correlation for heat transfer between the two ends. It was proportional to $(1+N)^{-1}$ in the boundary layer regime where the boundary layer thickness is smaller than the half width of each cell. Acharya and Tsang [3] studied inclined enclosures with a centrally located partition using a finite difference procedure. Chen *et al.* [8] studied numerically inclined partitioned enclosures of solar collectors and considered the heat conduction through the partitions. The range of Rayleigh numbers investigated was up to 5×10^4.

There are however several practical cases, such as composite solar collector systems [9] with several off-center partitions having finite thickness and finite conductivity and operating in both non-boundary layer and boundary layer regimes. The magnitudes of the ratio of conductivities of practical materials to that of air are about 10^0 for insulating type of materials, 10^1–10^2 for glass, various woods, gypsum plates, concrete, brick and 10^3–10^4 for metals. These cases are not considered in the literature and it is difficult to infer necessary information.

PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

The schematic of the system with the coordinates and boundary conditions is shown in Fig. 1. The vertical enclosure with multiple partitions of finite thickness and conductivity is bounded by two vertical isothermal walls at temperatures T_1 and T_2 and two horizontal adiabatic walls. The partitions are off-center and have a variable thickness l with identical conductivities. C, B, N and k_r are variable.

The mathematical model and the computer code were developed for a general case to study natural convection in cavities, which may be partially divided and may contain several walls [10]. The flow is assumed to be steady, laminar and two-dimensional.

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NOMENCLATURE

A	aspect ratio, H/W
b_{ij}	source term in the pressure correction equation
B	dimensionless cavity width, w/W
C	dimensionless partition thickness, l/W
c_p	heat capacity [$\mathrm{J\,kg^{-1}\,K^{-1}}$]
g	acceleration due to gravity [$\mathrm{m\,s^{-2}}$]
H	cavity height [m]
k	thermal conductivity [$\mathrm{W\,m^{-1}\,K^{-1}}$]
k_r	thermal conductivity ratio, k_w/k_f
l	partition thickness [m]
N	partition number
Nu	Nusselt number, equation (5)
p	pressure [Pa]
P	dimensionless pressure, $(p+\rho g y)/(\rho(\nu/W)^2)$
Pr	Prandtl number, ν/α
q	heat flux [$\mathrm{W\,m^{-2}}$]
Ra	Rayleigh number, $g\beta\Delta T W^3/(\nu\alpha)$
T	temperature [K]
ΔT	temperature difference between outer walls, T_2-T_1 [K]
u, v	fluid velocity in the x- and y-direction [$\mathrm{m\,s^{-1}}$]
U, V	dimensionless fluid velocities, uW/α, vW/α
w	width of the partitioned cavity [m]
W	cavity width [m]
x, y	Cartesian coordinates [m]
X, Y	dimensionless Cartesian coordinates, x/W, y/W.

Greek symbols

α	thermal diffusivity of fluid medium [$\mathrm{m^2\,s^{-1}}$]
β	thermal expansion of fluid [$\mathrm{K^{-1}}$]
Γ	diffusion coefficient
η	dimensionless thermal efficiency, $1-Nu_N/Nu_{N=0}$
Θ	dimensionless temperature, $(T-T_1)/\Delta T$
μ	dynamic viscosity [$\mathrm{kg\,m^{-1}\,s^{-1}}$]
ν	kinematic viscosity [$\mathrm{m^2\,s^{-1}}$]
ρ	fluid density [$\mathrm{kg\,m^{-3}}$].

Subscripts

1	left wall; cold wall; left channel
2	right wall; warm wall; right channel
f	fluid
w	wall.

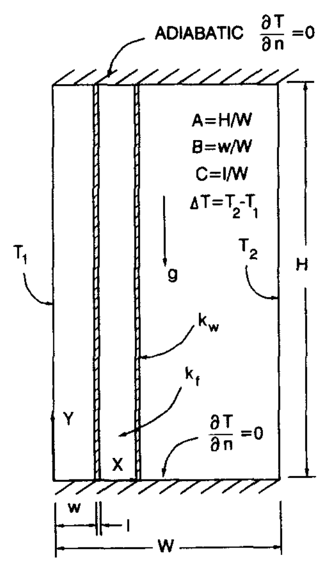

FIG. 1. Schematic of the problem, coordinate system and definition of geometrical parameters.

The Boussinesq approximation is used to account for the density variation. The non-dimensional continuity, momentum and energy equations are

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\Gamma\nabla^2 U \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\Gamma\nabla^2 V+Ra/Pr\cdot\Theta \tag{3}$$

$$U\frac{\partial \Theta}{\partial X}+V\frac{\partial \Theta}{\partial Y}=\frac{K}{Pr}\nabla^2\Theta \tag{4}$$

where $\Gamma = 1$ in the fluid region and 10^{15} in the solid region and $K = 1$ in the fluid region and k_r in the solid region. Γ and K are introduced in the discretized equations to ensure that the conduction in the partitions is accounted for and that $U = V = 0$ everywhere in the solid region including the solid–fluid interfaces.

The boundary conditions for this problem are as shown in Fig. 1. In addition, the no-slip conditions apply on all the rigid wall surfaces. The numerical method used to solve the system of equations (1)–(4) with these boundary conditions is the simpler method [11].

The heat transfer from the left and right bounding walls is calculated as

$$Nu = (1/A)\left[\int_0^A (\partial\Theta/\partial X)\,\mathrm{d}Y \Big/ \int_0^A (\partial\Theta/\partial X)\,\mathrm{d}Y|_{\mathrm{cond}}\right]. \tag{5}$$

VALIDATION OF THE CODE AND COMPUTATION

The computer code is validated with benchmark results in the literature and published elsewhere [10].

A uniform grid size was used. After several tries and comparisons with the test cases discussed above, it was found that depending on the dimensionless partition thickness C, a grid size of 24×30 to 26×30 ensured independence of solution on the grid size with reasonable computation time. For small Ra, the number of iterations was about 35. For larger Ra, the solution from the smaller Ra was used to initialize the computation so the number of iterations was reduced considerably. The relaxation parameter was varied from 0.80 to 0.70 for small Ra to about 0.40 for large Ra. The execution time for a typical case with $Ra = 10^5$ and 20 iterations was 12 CPUs on an IBM 3090.

The convergence criterion was based on the corrected pressure field. When the corrected terms were small enough so that no difference existed between the pressure field before and after correction, the computation was stopped. Therefore

$$\Sigma b_{ij} < 0.001. \tag{6}$$

In addition to the usual accuracy control, the accuracy of computations was controlled using the energy conservation within the system. It was found that it was accurate within 0.1% in the worst case.

RESULTS AND DISCUSSION

Various computations were carried out for C from 0.01 to 0.1, B from 0.1 to 0.9, N from 1 to 5 and k_r from 1 to 10^4 (with identical k_w for all partitions in each case and constant k_f for air) and Ra from 10^3 to 10^7. The fluid was air with $Pr = 0.72$. The results are represented to carry out parametric studies on the influence of various dimensionless parameters, namely A, B, C, N and k_r on Nu at various Ra. First, the case with $A = 5$ and $N = 2$ will be presented. Then, the effects of other parameters on the heat transfer will be discussed.

Case with $A = 5$ and $N = 2$

B, C and k_r are variables. Nu as a function of Ra is presented in Fig. 2 for the case of $A = 5$, $B = 0.2$, $C = 0.05$ and for various k_r. The relation of Nishimura *et al.* [6] for the case of diathermal partitions is also shown in the same figure for reference. This relation was obtained based on a boundary layer analysis and on the assumption of some empirical vertical temperature profiles of the partition plate and of the core region, which was verified with experiments. In their experimental study they used 10^{-4} m thick copper plate and water ($Pr = 7$) with C of about 0.005–0.01; k_r was about 20. The detailed results with $C = 0.01$ and k_r of this study (not shown here) have in fact confirmed that this relation represented well

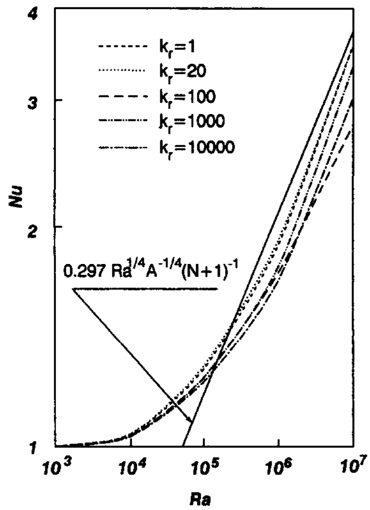

FIG. 2. Effect of k_r on heat transfer at various Ra. $A = 5$, $B = 0.2$, $C = 0.05$ and $N = 2$.

the case in consideration. Figure 2 shows that the equation of Nishimura *et al.* in the boundary layer regime represents the upper limit with diathermal partitions. For the conductive regime up to $Ra = 10^4$, the effect of k_r is negligible. For $Ra > 10^4$, Nu increases with increasing k_r as expected and it becomes a decreasing function of k_r when the latter is higher than about 20. This unexpected dependence of Nu on k_r is studied next.

Effect of k_r on Nu

The variation of Nu with k_r with $N = 2$ for various B and C at $Ra = 10^6$ is shown in Fig. 3 where it is seen that Nu passes from a maximum for a given k_r and for different C. To explain this, the stream function and isotherms with $B = 0.3$, $C = 0.05$ were plotted at $Ra = 10^6$ and for $k_r = 1$, 10 and 1000 (not shown

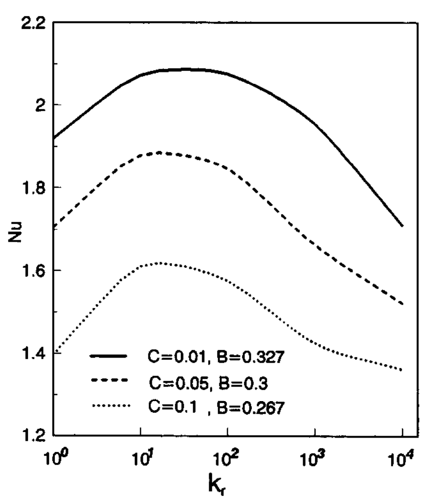

FIG. 3. Effect of k_r on heat transfer for various C and B. $A = 5$, $N = 2$ and $Ra = 10^6$.

here). It was seen that Ψ_{max} increased with increasing k_r, and that the reason for the observed phenomenon in Fig. 3 was not due to the strength of the convection. The heat transfer with $k_r = 1$ and 1000 is dominated more by conduction than that with $k_r = 10$. In the latter case, the isotherms undulated, which showed the effect of convection in the boundary layer regime. The temperature profiles at three elevations are shown in Fig. 4. For $k_r = 10$, the existence of the thermal boundary layers and the core regions are more visible than the other cases. It is also clear that the temperature gradients at the end walls and at the partitions are higher for the cases of $k_r = 10$ in comparison with smaller and larger k_r.

This phenomenon was studied for various other cases and it was seen that it was only the case wherever the convection heat transfer was a dominant mode. For example, with $A = 5$, $C = 0.1$, at $Ra = 10^3$ and 10^4 the effect was not observed. The effect was observed at $Ra = 10^5$, when $N > 2$, at $Ra = 10^6$, when $N > 3$, etc. To see the reason in more detail, a simpler case with $N = 1$ and $A = 5$, $B = 0.45$, $C = 0.1$ was studied. Figure 5 shows the variation of Nu as a function of k_r from 1 to 10^4 at various Ra. A maximum is not visible at Ra up to 10^4 (conductive regime) and Nu passes from a maximum at higher Ra. Values of k_r corresponding to these maximum Nu increase with increasing Ra. A comparison of the streamlines for various k_r showed that the center of the fluid circulation was shifted slightly upward for $k_r = 10$ and there was stratification in the left and right cavities. The fluid temperature profile was between those for $k_r = 1$ and 10^4 but close to that of the latter at the center and upper parts of the right cavity and those at the lower and center parts of the left cavity. However, the fluid temperature for $k_r = 10$ was much lower than the others in the lower part of the right

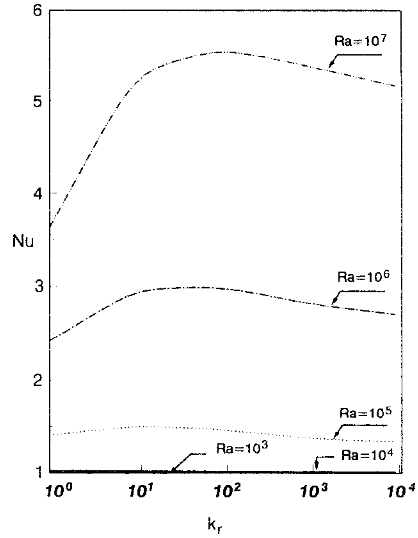

FIG. 5. Heat transfer as a function of k_r at various Rayleigh numbers. $A = 5$, $B = 0.45$, $C = 0.1$ and $N = 1$.

cavity and much higher at the upper part of the left cavity. That means the energy was generally transferred more effectively with $k_r = 10$ than that with 10^4.

Effects of C, B and A on Nu with N = 2

The effect of the partition thickness, C, with $k_r = 10^2$ on Nu is presented in Fig. 6 which shows two regimes for the influence of C on Nu. For low Ra, the dominant heat transfer is by conduction, the fluid layer occupies less space with increasing C and the heat transfer is dominated completely by conduction. At high Ra, the dominant heat transfer is by convection and the effect of the partition thickness is to reduce the heat transfer as expected. The effect of the partitioned cavity width, B, with $C = 0.05$ and $k_r = 1$,

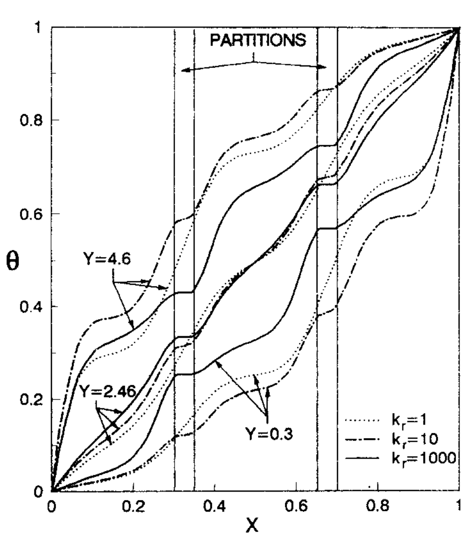

FIG. 4. Temperature profiles at three elevations (Y) for the case of $A = 5$, $B = 0.3$, $C = 0.05$, $N = 2$ and $Ra = 10^6$.

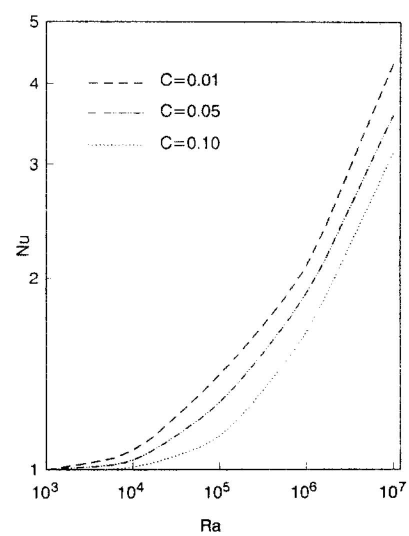

FIG. 6. Effect of C on heat transfer at various Ra. $A = 5$, $B = 0.2$, $N = 2$ and $k_r = 100$.

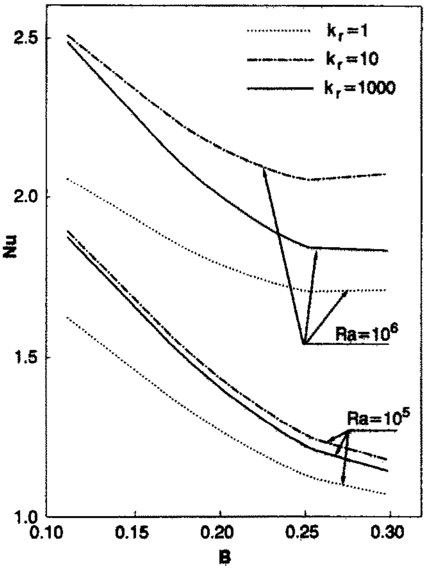

FIG. 7. Effect of B on heat transfer at various k_r and Ra. $A = 5$, $C = 0.05$ and $N = 2$.

10, 10^3 for $Ra = 10^5$ and 10^6 on heat transfer is presented in Fig. 7. The value of Nu decreases with increasing B, i.e. the off-center partitions are less effective in decreasing the heat transfer as reported earlier [2]. For $B = 0.3$, the cells are geometrically identical and the conduction is dominant. As B decreases, one of the cells becomes larger where the convection sets in and Nu increases. It is also seen that the heat transfer is higher with $k_r = 10$ than with others for the entire range of B. Figure 8 shows the effect of the aspect ratio, A, on the heat transfer at $Ra = 10^6$. As expected, the heat transfer decreases with increasing A for a given k_r. It shows also that the maximum heat transfer for k_r from 10 to 100 in Fig. 3 may be a phenomenon observed with small aspect ratios.

Effect of N on the heat transfer

The heat transfer for $N = 0$–5 is calculated for various k_r at Ra from 10^4 to 10^7 with $A = 5$, B and C variable. The results with $k_r = 10$ and $C = 0.05$ are presented in Fig. 9. The results of Bejan [12] with $N = 0$, i.e. the cavity without a partition, are also shown. The largest disagreement at $Ra = 10^4$ is about 16.5%, which may be acceptable in view of the large discrepancies in the literature (see, for example, ref. [13]). It is seen that at $Ra = 10^4$ the heat transfer is mostly by conduction and as the Rayleigh number increases, depending on the number of partitions, the convection heat transfer becomes evident. For $N = 1$, the dominant heat transfer is by convection at $Ra > 10^4$ whereas for $N = 5$, the heat transfer is by conduction up to about $Ra = 10^6$ after that the convection heat transfer is visible. Between these two limiting values of N the threshold Ra for the domination of convection heat transfer increases with increasing partition number. It can also be noticed that the heat transfer decreases with increasing N as already established in the literature for diathermal multiple partitions [6].

Efficiency of partitions

The reduction in heat transfer is defined as

$$\eta = 1 - Nu_N / Nu_{N=0}. \qquad (7)$$

Figure 10 shows the efficiency as a function of the partition number for $k_r = 1$ and 100, $C = 0.05$, $A = 5$ at $Ra = 10^6$ and 10^7. The efficiency is generally an increasing function of N and of k_r for $C = 0.05$ where the natural convection is a dominant heat transfer mode. For $C = 0.1$, the results showed (not presented here) that the conduction in the partitions and in

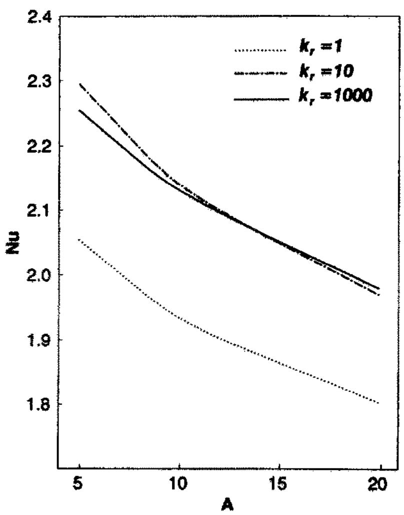

FIG. 8. Effect of A on heat transfer for various k_r. $B = 0.1$, $C = 0.05$, $N = 2$ and $Ra = 10^6$.

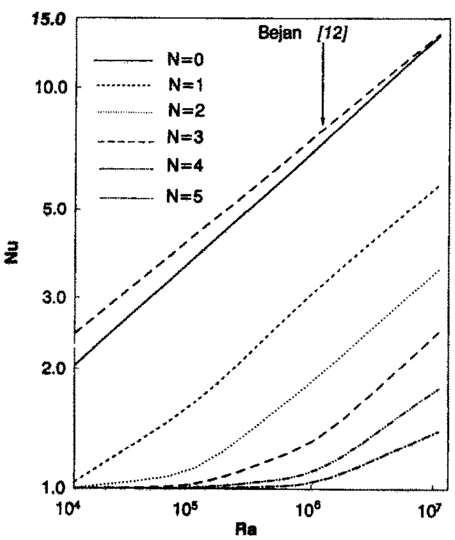

FIG. 9. Heat transfer as a function of Ra for various partition numbers. $A = 5$, $C = 0.05$ and $k_r = 10$.

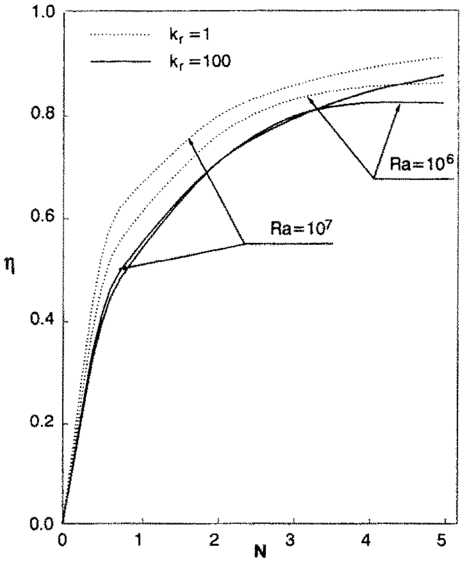

FIG. 10. Efficiency of partitions as a function of partition number and for various k_r and Ra. $A = 5$ and $C = 0.05$.

smaller cavities increasingly became more effective, and the efficiency passed from a maximum at a given N for each k_r except $k_r = 1$.

CONCLUSIONS

It is found that the heat transfer decreases with increasing partition number N at high Ra for all conductivity ratios k_r. It is dominated by conduction up to a threshold Ra above which the effect of convection starts. Although it is generally an increasing function of k_r, it passes however from a maximum when dominated by conduction. In this case, there is an optimum k_r for a maximum heat transfer across the cavity.

The heat transfer decreases with increasing partition thickness C at all Ra except in the conduction regime where the effect is negligibly small. Generally, the off-center partitions are less effective in decreasing the heat transfer. Nu is also a decreasing function of the cavity aspect ratio A.

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REFERENCES

1. H. Nakamura, Y. Asaka and T. Hirata, Natural convection and thermal radiation in enclosures with a partition plate, *Trans. J.S.M.E.* **B50**, 2647–2654 (1984).
2. T. W. Tong and F. M. Gerner, Natural convection in partitioned air-filled rectangular enclosures, *Int. Commun. Heat Mass Transfer* **13**, 99–108 (1986).
3. S. Acharya and C. H. Tsang, Natural convection in a fully partitioned, inclined enclosure, *Numer. Heat Transfer* **8**, 407–428 (1985).
4. R. Anderson and A. Bejan, Heat transfer through single and double vertical walls in natural convection: theory and experiment, *J. Heat Transfer* **24**, 1611–1620 (1981).
5. I. P. Jones, Numerical predictions from the IOTA 2 code for natural convection in vertical cavities, ASME Paper No. 82-HT-70 (1980).
6. T. Nishimura, M. Shiraishi, F. Nagasawa and Y. Kawamura, Natural convection heat transfer in enclosures with multiple vertical partitions, *Int. J. Heat Mass Transfer* **31**, 1679–1686 (1988).
7. T. Nishimura, M. Shiraishi and Y. Kawamura, Natural convection heat transfer in enclosures with an off-center partition, *Int. J. Heat Mass Transfer* **30**, 1756–1758 (1987).
8. W.-M. Chen, H. J. Shaw and M. J. Huang, Natural convection in partitioned enclosed spaces of solar collector, *Wärme- und Stoffübertr.* **25**, 59–67 (1990).
9. Z. Zrikem and E. Bilgen, Theoretical study of a composite Trombe–Michel wall solar collector system, *Solar Energy* **39**, 409–419 (1987).
10. R. Ben Yedder, Z.-G. Du and E. Bilgen, Numerical study of laminar natural convection in composite Trombe wall systems, *Solar Wind Technol.* **7**(6), 675–683 (1990).
11. S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*, p. 195. Hemisphere, Washington, DC (1980).
12. A. Bejan, Note on Gill's solution for free convection in a vertical enclosure, *J. Fluid Mech.* **90**, 561–568 (1979).
13. A. Bejan, *Convection Heat Transfer*. Wiley, New York (1986).

CONVECTION NATURELLE ET CONDUCTION DANS DES CAVITES AVEC PLUSIEURS CLOISONS VERTICALES

Résumé—On étudie théoriquement la convection naturelle laminaire et la conduction dans des cavités ayant plusieurs cloisons à épaisseur finie et conductrices. Les équations sont résolues en utilisant la formulation aux différences finies et la méthode du volume de contrôle. L'étude couvre Ra entre 10^3 et 10^7, $Pr = 0{,}72$ (air), rapport de forme $A = H/W$ entre 5 et 20, $B = w/W$ de 0,1 à 0,9 et $C = l/W$ de 0,01 à 0,1. Le nombre de partitionnement varie de 1 à 5 et le rapport des conductivités thermiques de la cloison et du fluide k_r de 1 à 10^4. Les résultats sont réduits en terme de Nu qui est fonction de Ra et des différents autres paramètres.

NATÜRLICHE KONVEKTION UND WÄRMELEITUNG IN HOHLRÄUMEN MIT MEHRFACHER SENKRECHTER UNTERTEILUNG

Zusammenfassung—Die laminare natürliche Konvektion und die Wärmeleitung in Hohlräumen mit mehrfachen Unterteilungen endlicher Dicke und Leitfähigkeit werden theoretisch untersucht. Die grundlegenden Erhaltungsgleichungen werden mit Hilfe eines Finite-Differenzen-Verfahrens mit Kontrollvolumina gelöst. Die Untersuchung deckt folgende Parameterbereiche ab: $10^3 < Ra < 10^7$; $Pr = 0{,}72$ (Luft); Seitenverhältnis $A = H/W$ im Bereich 5–20; $B = w/W$ im Bereich von 0,1–0,9 und $C = l/W$ im Bereich von 0,01–0,1. Die Zahl N der Unterteilungen wird von 1–5 variiert, das Verhältnis der Wärmeleitfähigkeiten von Unterteilung und Fluid liegt im Bereich $1 < k_r < 10^4$. Die Ergebnisse werden als Korrelationsgleichung zwischen Nu und Ra sowie weiteren Parametern angegeben.

ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ И ТЕПЛОПРОВОДНОСТЬ В ПОЛОСТЯХ С МНОЖЕСТВЕННЫМИ ВЕРТИКАЛЬНЫМИ ПЕРЕГОРОДКАМИ

Аннотация—Теоретически исследуются ламинарная естественная конвекция и передача тепла теплопроводностью в полостях с множественными перегородками, имеющими конечную толщину и теплопроводность. Уравнения решастся методом конечных разностей. Рассматривались числа Рэлея Ra, изменяющиеся от 10^3 до 10^7, $Pr = 0{,}72$ (воздух), отношение сторон $A = H/W$ изменялось от 5 до 20, $B = w/W$—от 0,1 до 0,9, $C = l/W$—от 0,01 до 0,1. Число перегородок N варьировалось от 1 до 5, а отношение коэффициентов теплопроводности перегородки и жидкости k_r—от 1 до 10^4.

Полученные результаты выражены в виде зависимозти от Ra и других параметров.